

## Studies on Difference Patterns from Cosecant Patterns

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**Abstract:** Array antennas are popular in radar and communication applications. It is possible to control the pattern characteristics from array antennas very effectively and easily. Array designers used different techniques to produce narrow beams. But it is essential to produce not only narrow beams but also other beam shapes like Square and Cosecant beams. These shaped beams are extremely useful in search, track radars and also for ground mapping. The difference patterns are often produced from sum patterns with additional phase, but in the case of marine radars either sum or difference patterns are asymmetrical in their sidelobe structure. In view of the above facts intensive investigations are carried out in the present work to generate asymmetrical difference patterns from Cosecant patterns with Fourier series, Fourier Transform and Woodward-Lawson methodologies.

**Keywords:** Cosecant beams, Difference Patterns, Fourier series, Fourier Transform, Woodward-Lawson method.

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### I. Introduction

In many applications it is desirable to have a single antenna, which is capable of radiating a pattern consisting of a narrow main beam and low sidelobes (often called the sum pattern) but also a pattern consisting of two principal lobes plus ancillary sidelobes (usually called the difference pattern). The sum pattern is produced from a directional antenna and the difference pattern is created by simultaneously exciting directional and Omni-directional antennas. In difference patterns, the depth of the null is significant and the difference slope is not enough for many applications. The difference patterns are used to determine the position of a target in a right/left and up/ down sense by pattern pointing, which places the target in the null between the twin lobes of difference pattern.

In fact, the combinations of sum and difference patterns have potential applications in acquisition and radar tracking systems. The sum pattern is responsible for range detection of moving targets and difference pattern determine the angular tracking accuracy. The sum patterns are generated by several methods, but the methods of generation of difference patterns are limited. A difference pattern typically has a null at broadside; these patterns are used for accurate determination of angle of arrival. The widely used monopulse radar antenna employs a difference pattern [1-2]. The aperture distributions considered here are those with a bi-phase distribution, where one-half of the aperture is in-phase, while the other half is  $180^\circ$  out of phase. The uniformly excited linear array, with the two halves of the array fed through a  $180^\circ$  hybrid, is a simple linear difference antenna.

To minimize the interference signals coming from jammers, stationary clutters, or other environmental disturbance, pattern nulling or pattern steering is required. Jama Mohamed et al. [3] proposed a method of simultaneous wide angle nulling for difference and sum beam patterns, to achieve required nulls in a specified direction. He also extended the work for minimization of gating lobes in Sum and difference patterns.

In height finding applications, the antennas also generate sharp beams for obtaining precise elevation information. If the beam is stationary in azimuth, an airplane flying across the beam will be illuminated for a period proportion to its distance away. To increase the time of illumination on nearby objects, a low intensity broad azimuth beam is required. If a fixed minimum illumination is to be achieved at a given linear distance, on both sides of central line of azimuth beam, a radiation pattern takes the shape of Cosecant beam. In fact, these beams are also popular for airport surveillance and ground mapping. The flat and Cosecant beams are considered to be shaped patterns which are different from narrow beams. For low scan rates, shaped reflectors are used to generate these patterns. Although these antennas are conceptually and structurally simple, flexibility is limited for the antenna designer [4-5].

Shaped beams are commonly used in base stations for uniform coverage, low interference and reduced electromagnetic pollution. The preferred vertical pattern shape for surface-based interrogators is a sector pattern or a cosecant-squared pattern (CSP). The CSP is preferred for long-range systems requiring high gain near the horizon. Search radars often employ a cosecant-squared antenna pattern. The pattern restricts the power at high-elevation angles, since an aircraft at a high-elevation angle is necessarily at close range and little power is

required for detection. Owing to reducing multipath effects and minimizing errors in detection, low sidelobe level (SLL) and low ripple at the shaped-beam region are necessary [6-7].

Woodward and Lawson introduced a very popular antenna pattern synthesis method used for beam shaping. The synthesis is accomplished by sampling the desired pattern at various discrete locations. Each pattern sample is associated with a harmonic current of uniform amplitude distribution and uniform progressive phase, whose corresponding field is known as a composing function. Woodward-Lawson synthesis technique reconstructs pattern whose values at the sampled points are similar to the ones of the desired signal, but it is unable to control the pattern between the sample points [8-9].

The Woodward-Lawson method tend to produce low sidelobes and low main beam ripple at some sacrifice in transition width. On the other hand, Fourier methods yield somewhat inferior side-lobe levels and ripples. The Fourier series synthesized pattern gives very sharp roll-off from the main beam to the side-lobe region; that is, small transition width. The Fourier transform synthesized pattern yields the least mean-square error (MSE) or least mean-squared deviation from the desired pattern.

It is evident from the above discussion no researcher has reported intensive data related to difference patterns from Cosecant patterns. In this paper, to optimize these patterns the conventional methods like Fourier transform Fourier series, Woodward Lawson are proposed.

## II. Formulation

The radiation field of the symmetric broadside array of  $2N+1$  isotropic element is given by

$$E(u) = a_0 + 2 \sum_{i=1}^N a_n \cos u_n \quad (2.1)$$

$$E(u) = a_0 + 2 \sum_{i=1}^N a_0 \cos(a_0 \beta d_0 u) \quad (2.2)$$

Here,  $u_n = \beta d_n \sin \theta$   
 $u = \sin \theta$

It is possible to replace dual antennas namely a directional and monopole antennas by dual excitations. It is required to phase the antenna to obtain both sum and difference patterns.

The difference pattern is given by

$$E_d(u) = 4 \sum_{i=1}^N a_n \sin(b_n \beta d_N u_s) \sin(a_n \beta d_N u) \quad (2.3)$$

In the above expression,

$$u_s = \sin \theta_s$$

$\theta_s$  is Squint angle of the beam.

The difference pattern from a continuous line source is also obtained from

$$E_d(u) = \left[ \int_{-1}^0 A(x) e^{j \frac{2\pi L}{\lambda} [ux+\phi]} dx + \int_0^1 A(x) e^{j \frac{2\pi L}{\lambda} [ux+\phi]} dx \right] \quad (2.4)$$

Here  $A(x)$  is excitation function.

To generate a null in the boresight directions  $180^\circ$  phase shift is introduced to one half of the array i.e.

$$\phi = 0 \text{ for } -1 \leq x \leq 0$$

$$\phi = \pi \text{ for } 0 \leq x \leq 1 \quad (2.5)$$

Substituting equation (2.5) in equation (2.4), then the radiation pattern is given by

$$E_d(\mathbf{u}) = \left[ \int_{-1}^0 A(\mathbf{x}) e^{j\frac{2\pi L}{\lambda} u \mathbf{x}} d\mathbf{x} + \int_0^1 A(\mathbf{x}) e^{j\frac{2\pi L}{\lambda} [u \mathbf{x}_n + \pi]} d\mathbf{x} \right] \quad (2.6)$$

The difference pattern for a discrete array is given by

$$E_d(\mathbf{u}) = \left[ \sum_{n=1}^{N/2} A(\mathbf{x}_n) e^{j\frac{2\pi L}{\lambda} u \mathbf{x}_n} + \sum_{n=\frac{N}{2}+1}^N A(\mathbf{x}_n) e^{j\frac{2\pi L}{\lambda} [u \mathbf{x}_n + \phi(\mathbf{x}_n)]} \right] \quad (2.7)$$

Here,

- $A(\mathbf{x}_n)$  = Amplitude distribution
- $\phi(\mathbf{x}_n)$  = Excitation phase distribution
- $\frac{2L}{\lambda}$  = Normalized array length
- $u$  =  $\sin\theta$ ,  $\theta$  is the angle of observer and broadside
- $\mathbf{x}_n$  = Spacing function =  $\frac{2n - N - 1}{N}$
- $N$  = Number of elements

## 2.1. Generation of cosecant arrays with difference patterns by different shaped beam synthesis methods

### 2.1.1 Woodward Lawson Method:

The Woodward Lawson sampling method for line sources is an extendable method for linear arrays. Here the elements are placed equally with specified spacing. The array approximation to the continuous case is made by sampling the current distribution (5.8) at the array element positions and element currents are given by

$$A(\mathbf{x}) = \frac{1}{N} b_n e^{jkx_n \cos \theta_n}$$

The normalized amplitude distribution of the array is obtained from

$$A(\mathbf{x}_n) = \sum_{n=-M}^M \frac{1}{N} b_n e^{jkx_n \cos \theta_n} \quad (2.8)$$

Associated with each current source, the corresponding field pattern is

$$f_n(\mathbf{u}) = b_n \frac{\sin \frac{N}{2} kd(\mathbf{u} - \mathbf{u}_n)}{N \sin \frac{1}{2} kd(\mathbf{u} - \mathbf{u}_n)}$$

Where  $\mathbf{u} = \cos \theta$

$$\mathbf{u}_n = \cos \left( \frac{n\lambda}{Nd} \right) \quad |n| \leq M$$

$$E(u) = \sum_{n=-m}^m b_n \frac{\sin \frac{N}{2} kd(u - u_n)}{N \sin \frac{1}{2} kd(u - u_n)} \quad (2.9)$$

Where  $b_n$  is excitation coefficient of each element of the array at its location, equal to the value of the specified pattern at the sample points

$$b_n = E(u = u_n)_d \quad (2.10)$$

### 2.1.2 Fourier Series Method

We can obtain good approximations by sampling the line-source distribution with an array. Beyond sampling a line source, we can also apply Fourier series to design an array directly. An array for a shaped beam must be much larger than is required for the beamwidth. The extra size of the array gives us the degrees of freedom necessary for beam shaping. Increasing the array size increases the match between the specified and the actual beam shape.

The desired pattern function  $E_d(u)$ , can be expanded into a Fourier series in the interval  $-\lambda/2d < u < \lambda/2d$

$$E_d(u) = \sum_{n=-\infty}^{\infty} b_n e^{j2\pi n(d/\lambda)u} \quad (2.11)$$

Integrating on both sides

$$\begin{aligned} \int_{-\lambda/2d}^{\lambda/2d} E_d(u) e^{j2\pi n(d/\lambda)u} du &= \sum_{n=-\infty}^{\infty} b_n \int_{-\lambda/2d}^{\lambda/2d} e^{j2\pi(m-n)(d/\lambda)u} du \\ &= \sum_{n=-\infty}^{\infty} b_n \frac{e^{j2\pi(m-n)(d/\lambda)\lambda/2d} - e^{-j2\pi(m-n)(d/\lambda)\lambda/2d}}{j2\pi(m-n)(d/\lambda)} \\ &= \sum_{n=-\infty}^{\infty} b_n \frac{2j \sin[\pi(m-n)]}{2j\pi(m-n)d/\lambda} \\ &= \frac{\lambda}{d} \sum_{n=-\infty}^{\infty} b_n \frac{\sin[\pi(m-n)]}{\pi(m-n)} = \frac{\lambda}{d} b_m \end{aligned}$$

$$b_n = \frac{d}{\lambda} \int_{-\lambda/2d}^{\lambda/2d} E_d(u) e^{-j2\pi n(d/\lambda)u} du \quad (2.12)$$

Here,

$d$  is the spacing between the elements

$u = \sin\theta$

$\theta$  is the angle from the broad side of the array

The normalized amplitude distribution of the array is therefore, obtained from

$$b_n = \frac{d}{\lambda} \int_{-\lambda/2d}^{\lambda/2d} E_d(u) e^{-j2\pi n(d/\lambda)u} du \quad (2.13)$$

An infinite array is of course, not practical, but truncating the series (2.12) to a finite number of terms produces the following approximation to  $E_d(u)$

$$E_d(u) = \sum_{n=-N}^N b_n e^{j2\pi n(d/\lambda)u} \tag{2.14}$$

If we let the currents of each element in the array equal the Fourier series coefficients, that is,

$$A_n = b_n, \quad |n| \leq N \tag{2.15}$$

The Fourier series synthesis procedure is, then, to use element excitations  $A_n$  equal to the Fourier series coefficients  $b_n$  calculated from the desired pattern.

**2.1.3 Fourier Transform Method:**

The Fourier transform pair relationship suggests a synthesis method for the desired pattern and its current distribution. If  $E_d(u)$  is the desired pattern, the corresponding current distribution  $A_d(x)$  is found easily from  $x = 1, 2, 3, \dots, N, j = \sqrt{-1}$

$$A_d(x) = \int_{-\infty}^{\infty} E_d(u) e^{-j2\pi ux} du \tag{2.16}$$

This is very direct, but unfortunately, the resulting  $A_d(x)$  will not be confined to  $|n| \leq L/2\lambda$  as required, it will be in fact of infinite extent. An approximate solution can be obtained by truncating  $A_d(x)$ , giving the synthesized current distribution as follows:

$$A(x) = \begin{cases} A_d(x), & |n| \leq L/2\lambda \\ 0, & |n| > L/2\lambda \end{cases} \tag{2.17}$$

For pattern calculation

$$E(u) = \sum_{n=1}^N A(x_n) e^{j2\pi x_n u} \tag{2.18}$$

For all the above three traditional synthesis methods, to get the current distribution by taking the desired pattern function from expression (2.19).

The desired pattern function  $E_d(u)$  can be defined as

$$E_d(u) = \begin{cases} 0, & 0 \leq u \leq u_0 \\ \frac{u_0}{u}, & u_0 \leq u \leq u_1 \end{cases} \tag{2.19}$$

### III. Results

By introducing the current distribution equations (2.8), (2.14) and (2.16) in equation (2.7) to get the difference patterns from Cosecant arrays. Using the equation (2.7), the radiation patterns of discrete arrays are computed. The patterns are computed numerically and presented in figures (3.1-3.5).

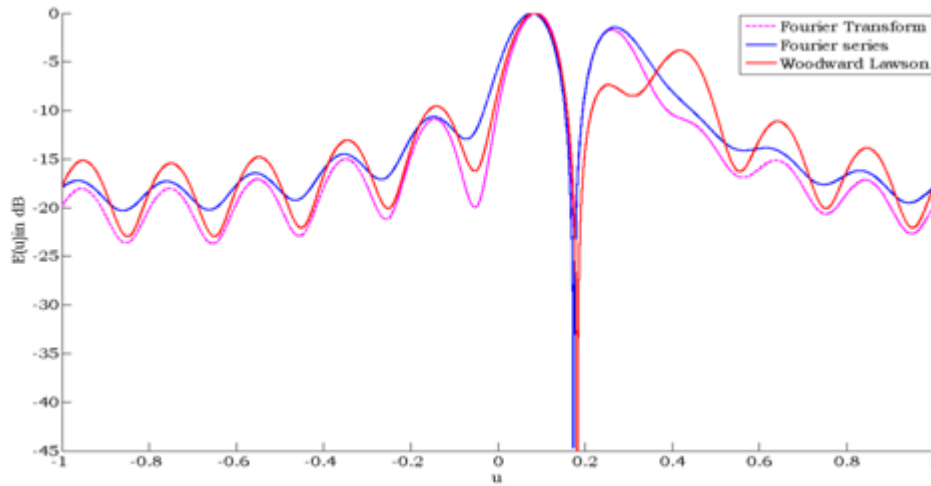


Fig. 3.1 Difference patterns from Cosecant array of elements for N=20

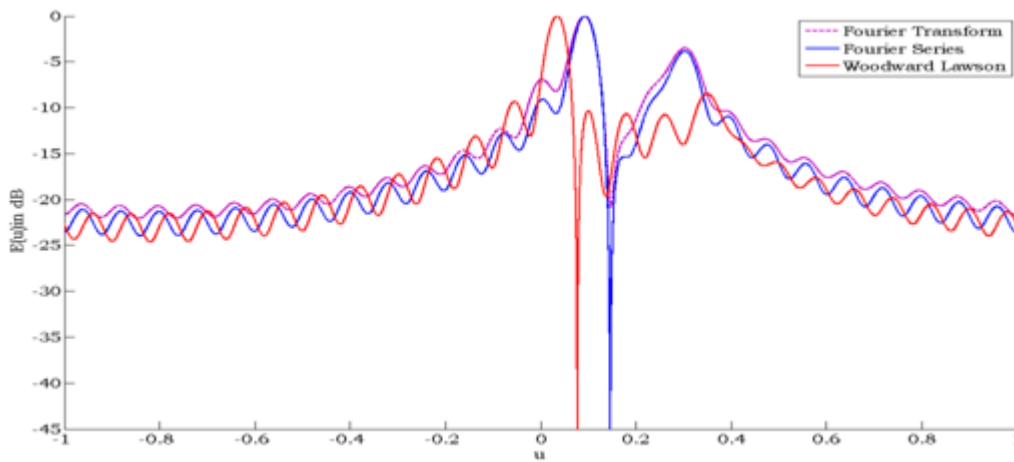


Fig. 3.2 Difference patterns from Cosecant array of elements for N=50

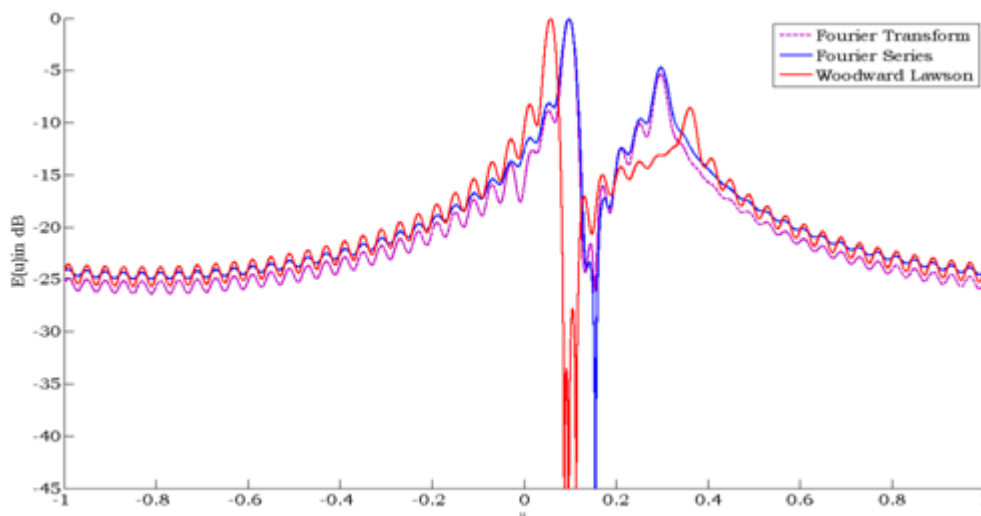


Fig. 3.3 Difference patterns from Cosecant array of elements for N=100

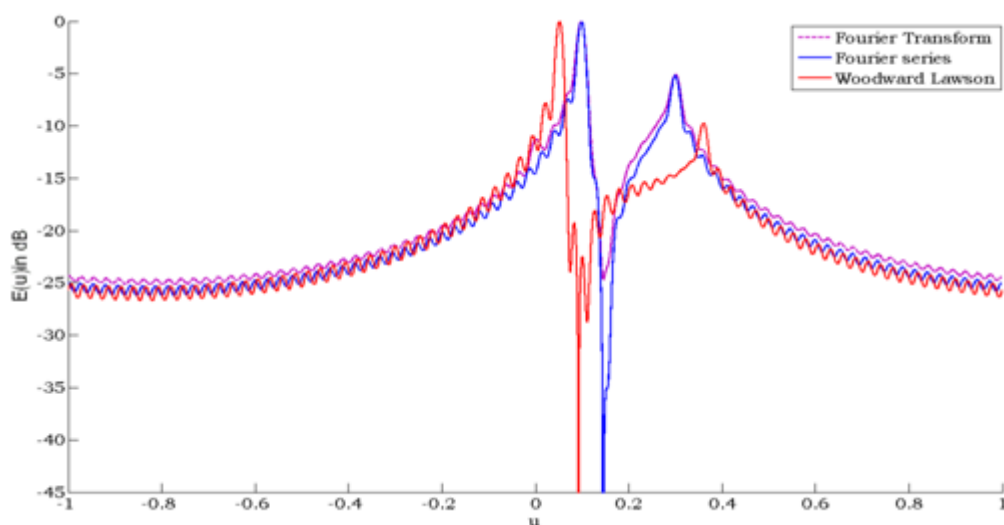


Fig. 3.4 Difference patterns from Cosecant array of elements for N=150

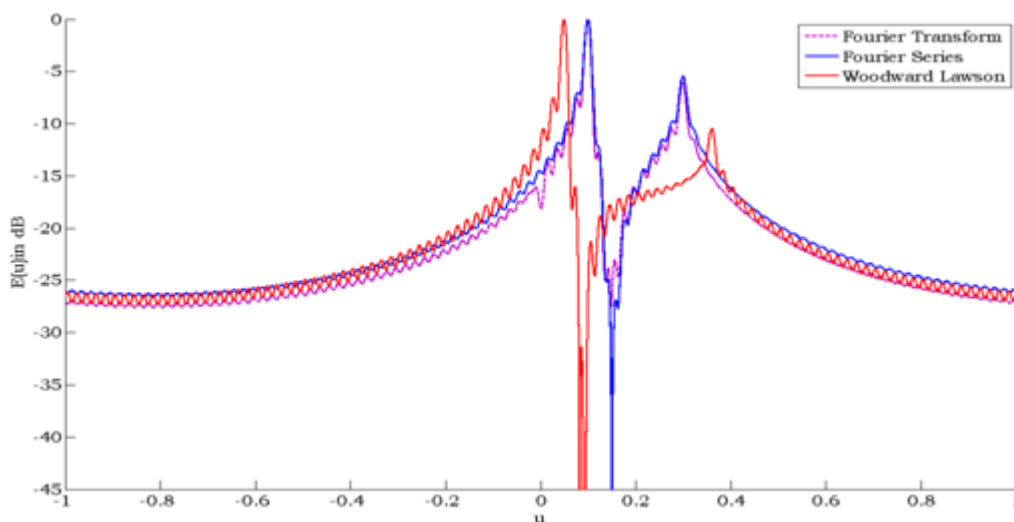


Fig. 3.5 Difference patterns from Cosecant array of elements for N=200

#### IV. Conclusions

The difference patterns computed to have deep null which is right side of the boresight. The number of lobes including minor one is found to increase with increasing the number of elements. The width of the difference lobes is small for large and large for small arrays. The amplitude distributions are tapered symmetrical ones, the phase distributions are asymmetrical and the spacing is uniform. Here Ishimaru's spacing function is used.

The Woodward-Lawson method produce low sidelobes and low main beam ripple, but as the number of elements are increased the difference pattern have more number of sidelobe ripples. The Fourier series synthesized pattern gives very sharp roll-off from the main beam to the side-lobe region; that is, small transition width. The Fourier transform synthesized patterns have the least mean-squared deviation from the desired pattern. These types of patterns are used in height finding applications, air surveillance and especially used in marine radars where pitch and roll of the ship exists.

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### Author's Profile



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